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|------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1 sample Z                                     | $z = \frac{\bar{x} - \mu_0}{(\sigma / \sqrt{n})}$                                                                                                                                                                                                                       | $x_i$ normal or $n > "30"$                                                                                                                                                                     |
| 2 sample Z                                     | $z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \stackrel{H_0}{=} \frac{\Delta \bar{x}}{\sigma_{\Delta \bar{x}}}$                                                                                         | Normal, independent, $\sigma_i$ known but unequal.<br>Note: $H_0 : \Delta \mu = 0$                                                                                                             |
| 1 sample t-test                                | $t = \frac{\bar{x} - \mu_0}{(s / \sqrt{n})} \quad df = n - 1$                                                                                                                                                                                                           | Normal population or $n > 36$ , $\sigma_i$ unknown                                                                                                                                             |
| Paired t-test                                  | $t = \frac{\bar{d} - d_0}{(s_d / \sqrt{n})} \quad df = n - 1$                                                                                                                                                                                                           | Normal population of <i>PAIRED</i> differences $\sigma_i$ unknown (or large n);<br>$d_0$ = population mean difference                                                                          |
| 1-proportion z-test                            | $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$                                                                                                                                                                                                                    | $np, nq > "7"$ for normal approx, SRS (Simple Random Sample)<br>$q_0 = 1 - p_0$                                                                                                                |
| 2-proportion z-test, equal variances           | $z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$<br>$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$                                                                                                    | $np, nq > 7$ , independent obs.<br>$H_0 : \Delta p = 0$                                                                                                                                        |
| 2-sample t-test, Unequal Variances             | $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$<br>$df = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\left(\frac{S_1^2}{n_1}\right)^2 / (n_1 - 1) + \left(\frac{S_2^2}{n_2}\right)^2 / (n_2 - 1)}$ | Normal population or $n_1 + n_2 > 43$ , independent obs.,<br>$H_0 : \Delta \mu = 0$<br>Note in this case $df \leq n_1 + n_2 - 2$<br>"Desert Island" rule-of-thumb:<br>$\min(n_1 - 1, n_2 - 1)$ |
| 2-sample pooled t-test                         | $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$<br>$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$<br>$df = n_1 + n_2 - 2$                                                                             | Normal population or $n_1 + n_2 > 43$ , independent obs., $\sigma_1 = \sigma_2$<br>$H_0 : \Delta \mu = 0$                                                                                      |
| Test for variance/standard deviation, X Normal | $\frac{(n-1)s^2}{\sigma^2} \sim \chi_{n-1}^2, \text{ so } \Pr(a \leq \frac{(n-1)s^2}{\sigma^2} \leq b) = \Pr\left(\frac{(n-1)s^2}{b} \leq \sigma^2 \leq \frac{(n-1)s^2}{a}\right)$                                                                                      | Normal population or N large and f(x) largely symmetric;<br>$H_0 : \sigma^2 = \sigma_0^2$ ; a < b chosen such that $P(a \leq \chi_{n-1}^2 \leq b) = 1 - \alpha$                                |

|                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                       |
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| Test for unity ratio of variances/standard deviations, $X_i$ Normal | Under $H_0$ , $\frac{S_1^2 / \sigma_1^2}{S_2^2 / \sigma_2^2} = \frac{S_1^2}{S_2^2} = \left( \frac{S_1}{S_2} \right)^2 \sim F_{n_1-1, n_2-1}$                                                                                                                                                                                                                                                                                      | Normal population or N large and $f(x)$ largely symmetric;<br>$H_0 : \sigma_1^2 = \sigma_2^2$<br>Use properties of $F$ distribution and put largest variance in numerator; $p$ -value is twice the $p$ -value of the 1-sided $\frac{\alpha}{2}$ test. |
| Test for Zero correlation                                           | Under $H_0$ , $\sqrt{\frac{(n-2)r^2}{1-r^2}} \approx t_{n-2}$                                                                                                                                                                                                                                                                                                                                                                     | Normal population.<br>$H_0 : \rho = 0$<br>$H_1 : \rho \neq 0$                                                                                                                                                                                         |
| Test for general correlation                                        | Fisher's Z-Transform (to normality)<br>$z' = \tanh^{-1}(r) = \frac{1}{2} \ln \left( \frac{1+r}{1-r} \right) \stackrel{d}{\approx} N \left( \operatorname{atanh}(\rho), \frac{1}{n-3} \right)$<br>$\xi_l = z' - z_{\frac{\alpha}{2}} \sigma_{z'}; \xi_u = z' + z_{\frac{\alpha}{2}} \sigma_{z'}$<br>$\rho_l = \tanh(\xi_l) = \frac{e^{2\xi_l} - 1}{e^{2\xi_l} + 1}; \rho_u = \tanh(\xi_u) = \frac{e^{2\xi_u} - 1}{e^{2\xi_u} + 1}$ | Bivariate normal population<br>Bias is $\frac{2\rho}{n-1}$<br>$\sigma_{z'}^2 = \frac{1}{n-3}$ , ind of $r$                                                                                                                                            |
| Wilks Test (Asymptotic Likelihood Ratio test)                       | $T_n = 2 \left( l(\hat{\theta}   x) - l(\hat{\theta}_0   x) \right) \xrightarrow{d} \chi_\nu^2$<br>Rejection region $R = \{x : T_n > \chi_{\nu, \alpha}^2\}$                                                                                                                                                                                                                                                                      | $l$ is log-likelihood function;<br>$\hat{\theta}, \hat{\theta}_0$ are the unrestricted/restricted mle, respectively. $\nu$ is number of free parameters specified in null.<br>Likely most accurate for moderate $n$ .                                 |
| Wald Test                                                           | $W_n = \frac{\hat{\theta}_n - \theta_0}{\sqrt{I_n(\theta_0)}} \xrightarrow{d} N(0,1)$<br>$(1-\alpha)CI : \hat{\theta}_n \pm z_{\frac{\alpha}{2}} \cdot I_n^{-\frac{1}{2}}(\hat{\theta}_n)$<br>$n(\hat{\theta}_n - \theta_0)^T I_1(\hat{\theta}_n)(\hat{\theta}_n - \theta_0) \xrightarrow{d} \chi_k^2$                                                                                                                            | $\theta \in \Theta \subset R^k$ . Result holds under $H_0$ . $I_n(\theta) / J_n(\theta)$ is expected/observed Fisher information.                                                                                                                     |
| Rao (Score) Test                                                    | $U_n \approx N_k(0, nI_1(\theta_0))$ , or rather $\frac{U_n}{\sqrt{n}} \xrightarrow{d} N_k(0, I_1(\theta_0))$<br>$R_n = \frac{1}{n} U(\theta_0)^T I_1^{-1}(\theta_0) U(\theta_0) \xrightarrow{d} \chi_k^2$                                                                                                                                                                                                                        | $\theta \in \Theta \subset R^k$ ;<br>$U(\theta) = \frac{\partial}{\partial \theta} \ln f(\theta   x)$                                                                                                                                                 |

#### References.

Various

Anirban DasGupta, *Asymptotic Theory of Statistics and Probability*, Springer-Verlag New York, 2008. ISBN 978-0-387-75970-8